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Summer Internship Report, Summer 2011
September 1, 2011
Internship Advisor: Dr. Rudd
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This summer, I explored the use of functional equations in finding solutions to a degenerate nonlinear partial differential equation (PDE) with particular boundary data. I worked with Dr. Matthew Rudd, who recently wrote a paper asserting that this problem can actually by examined through methods much more elementary than those customarily used. This means that even an undergraduate (like me) can not only understand but also find solutions to this advanced math problem. This can be done using a combination of simple statistical measures, mean and median, in an iterative fashion. Our main goal was to support his rigorous proof with numerical examples, as well as to explore solutions of different boundary data and other variables, including those in the fourth dimension.

My work involved experimenting with different domains and parameters to discover various p-harmonic solutions, as well as creating graphics such as movies and pictures to watch the development of different functions through iterations. Using the computer programming language R, I did computational work, discussed the results with Dr. Rudd, and then returned to alter the code accordingly and continue experimenting. Preliminary work involved not only learning the basics of the math problem, but also learning a new computer programming language, since R is much more applicable for statistical problems like this than are other mathematical tools, such as Mathematica or Matlab. This

meant that my work was sometimes slow, but nonetheless the internship was incredibly rewarding, both for me personally and from the insight it gave into solutions of the partial differential equation. I intend to continue our research in an independent study, to present our findings at Scholarship Sewanee and in my senior mathematics talk, as well as to submit a paper to be published with Dr. Rudd based on our work, and possibly to speak at a math conference at another university with Dr. Rudd's supervision.

My internship this summer has furnished me with plenty of skills that I will no doubt use in later years. Simply learning how to use the statistical-based computer program R alone was a priceless skill I acquired. The program is perfect for any research that involves a large amount of numerical data. If I am given the chance to conduct research after Sewanee, whether it is in math, geology (my other major), or other departments that interest me such as ecology or natural resources, R will most definitely come in handy. And working with R gave me priceless experience in computer programming, which is another career possibility. Also, learning to solve a PDE is a good tool, especially if I decide to go to graduate school in mathematics, where I could further study differential equations and apply my knowledge of the solutions to this PDE. Possibly the most important thing I gained from this internship, however, was the chance to study math that went far beyond my comfort zone. I haven't had all the math necessary to fully understand the problem in particular or the theories behind it. To understand the theory entirely would most likely mean months of graduate level mathematics studies. So I had to use a limited knowledge of partial

differential equations to understand and solve the problem. While this meant that I frequently had to seek help from Dr. Rudd or others, it also meant that I improved greatly in my confidence to ask questions and accept that I may not totally comprehend the answer, which I think is essential to problem solving. Also, this means that there is still an endless amount of work that I can do in this field this year—something I'm very excited about.

The challenges that I encountered in the internship mostly involved programming in R. R itself, though computationally efficient and excellent for statistics work, is not especially user-friendly. Although I used books as well as online manuals, there were many times when a small glitch in my code took hours to pinpoint, or when I had trouble finding a specific method or tool to use when converting my ideas on paper to computer code. For example, throughout the seven weeks that I interned here, Dr. Rudd and I continually tried and failed to create a video of successive iterations in his algorithm. We tried a number of different video methods in R to produce a small moving graphic of the change from an initial function to its solution, until the last day. That day was arguably the most exciting, when I finally figured out how to create the video, which we have now completed and can use both as a teaching tool and as a way to observe how functions change over time. Also, it goes without saying that trying to understand a math problem that was beyond my ability level was often frustrating.

The math problem itself is not one that will likely be applicable to my career goal in particular, as I intend to pursue a career in geology or hydrology.

However, fields of geology utilize differential equations (e.g. analyzing groundwater flow or seismicity). The techniques that I learned with this problem may be applicable to research in geology. This problem was also very statistics-based. Any research I conduct in graduate school could possibly use similar analyzing and graphing techniques as those that I utilized to develop graphics for this problem. Finally, the most important thing accomplished with this project, I think, is the fact that Dr. Rudd has developed a priceless algorithm that can be used as a learning and teaching tool. If it can be used in this PDE, similar techniques could be used in further problems. Also, we did not explore the case of $p < 1$ for p-harmonic problems in great detail, and this technique opens the door for this family of problems to be explored by other math enthusiasts.

Finally, one of the most enjoyable parts of this internship was simply being part of Sewanee in the summertime. I got to enjoy this community at its quietest time of year, which was a great way to get in touch with the beauty of the Mountain and experience and even participate in activities outside the University. Overall, this internship was an amazing experience that I would recommend to any other student who likes math, sunshine, good music, and good friends.